Spherical Collapse with Dark Energy

Irit Maor

CERCA, Department of Physics, Case Western Reserve University,

*10900 Euclid Ave, Cleveland, OH 44106-7079 USA**

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I discuss the work of Maor and Lahav [1], in which the inclusion of dark energy into the spherical collapse formalism is reviewed. Adopting a phenomenological approach, I consider the consequences of - a) allowing the dark energy to cluster, and, b) including the dark energy in the virialization process. Both of these issues affect the final state of the system in a fundamental way. The results suggest a potentially differentiating signature between a true cosmological constant and a dynamic form of dark energy. This signature is unique in the sense that it does not depend on a measurement of the value of the equation of state of dark energy.

I. INTRODUCTION

One of the outstanding issues of cosmology is dark energy. The primary question is whether dark energy is a cosmological constant, or is it dynamical. In order to use inhomogeneity studies to probe dark energy, it is essential that we understand how the presence of dark energy affects the evolution of overdensities. Adopting a phenomenological approach, the aim of this work is to consider what are the effects on the evolution of inhomogeneities if the dark energy clusters, or, alternatively, if it participates in the virialization process. It is based on the work of Maor and Lahav [1].

A fundamental tool in the analysis of inhomogeneities is the spherical collapse formalism, which dates back to Gunn and Gott [2]. It describes how a small spherical patch of homogeneous over-density decouples from the expansion of the universe, slows down, and eventually turns around and collapses. It is assumed that the collapse is not complete, thus it does not lead into a singularity. Instead, the system eventually virializes and stabilizes,

*Electronic address: irit.maor@case.edu

having a finite size. The definition of the moment of virialization depends on energy considerations. The top hat spherical collapse is incorporated, for example, in the Press-Schecter [3] formalism. It is, therefore, widely used in present day interpretation of data sets.

The generalization of the spherical collapse formalism to include additional forms of energy has been subject to numerous studies [1, 4, 5, 6, 7, 8, 9, 10, 11, 12]. Lahav *et al* [4] generalized the formalism to a universe composed of ordinary matter and a cosmological constant. Wang and Steinhardt [5] included Quintessence with a constant or a slowly varying equation of state. Battye and Weller [8] included Quintessence in a different manner than Wang and Steinhardt, taking into account its pressure. Mota and Van de Bruck [10] considered spherical collapse for different potentials of the Quintessence field, and checked what happens when one relaxes the common assumption that the Quintessence field does not cluster on the relevant scales. Maor and Lahav [1] considered both clustering and homogeneous dark energy, and examined what are the effects if dark energy participates in the virialization process. They pointed out a source of energy non conservation in the case where dark energy is kept homogeneous, and suggested how to incorporate this energy non conservation. Wang [12] considered another source of energy non-conservation, due to the fact that a homogeneous dark energy acts as a time-dependent, and hence nonconservative force.

The structure of this paper is as follows. Section II reviews the basics of the spherical collapse formalism. The procedure by which we define virialization of an overdensity is reviewed in section III. For non clustering dark energy which is not a cosmological constant there are some problems regarding energy conservation, which are discussed in section IV. Section V presents some results, and section VI is dedicated to concluding remarks.

II. SPHERICAL COLLAPSE

We take the background cosmology to be a flat FRW universe with two energy components. One is non relativistic dust ρ_m with pressure $p_m = 0$ (for the sake of this discussion it is unimportant if this component is luminous or not). The second component is the dark energy, modelled as a perfect fluid with pressure $p_Q = w\rho_Q$, w being the (constant) equation of state. The equations governing the background evolution are then

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\left(\rho_m + \rho_Q\right) \tag{1}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho_m + (1+3w)\,\rho_Q\right) \tag{2}$$

$$\dot{\rho}_m + 3\left(\frac{\dot{a}}{a}\right)\rho_m = 0 \tag{3}$$

$$\dot{\rho}_Q + 3(1+w)\left(\frac{\dot{a}}{a}\right)\rho_Q = 0\ , \tag{4}$$

where a is the global scale factor.

Within such a universe, we assume that there is a spherical perturbation in the matter density, with a flat (top hat) profile. ρ_{mc} denotes the matter density within the perturbation. We assume that the initial perturbation is in the matter field only, though we will allow non-homogeneity to develop for the additional fluid. Following the spherical collapse formalism, the equations governing the evolution of the overdensity are similar to those of the background, with the global scale factor a replaced with the local scale factor R. The flatness condition is not held, because of the perturbation in the matter,

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3}\left(\rho_{mc} + (1+3w)\,\rho_{Qc}\right) \tag{5}$$

$$\dot{\rho}_{mc} + 3\left(\frac{\dot{R}}{R}\right)\rho_{mc} = 0 \tag{6}$$

$$\dot{\rho}_{Qc} + 3(1+w)\left(\frac{\dot{R}}{R}\right)\rho_{Qc} = \gamma\Gamma\ , \tag{7}$$

with

$$\Gamma = 3(1+w)\left(\frac{\dot{R}}{R} - \frac{\dot{a}}{a}\right)\rho_{Qc} \tag{8}$$

$$0 \le \gamma \le 1\ . \tag{9}$$

The form of equation (7) allows us to move, in a continuous way, between two cases of interest. The first case is where the dark energy is kept homogeneous, by choosing $\gamma = 1$. Equation (7) then reads $\dot{\rho}_{Qc} + 3(1+w)\left(\frac{\dot{a}}{a}\right)\rho_{Qc} = 0$, and, therefore, the evolution of the dark energy within the perturbation is similar to that of the dark energy in the background, $\rho_{Qc} = \rho_Q$. The second case of interest is where the dark energy is allowed to follow the local scale factor and fully cluster. This is done by taking $\gamma = 0$, in which case equation (7) reads $\dot{\rho}_{Qc} + 3(1+w)\left(\frac{\dot{R}}{R}\right)\rho_{Qc} = 0$. Thus one can think of γ as a clustering parameter. This new parameter introduces a new scale to the problem, which defines the clustering rate of the dark energy. $\gamma\Gamma$ is the rate in which the perturbation loses energy to the background, due to the Q field. The system is conservative when $\gamma = 0$ and the Q field is allowed to fully cluster, or when $\Gamma = 0$. The latter case corresponds to $w = -1$. In all other cases, the

system loses energy to the background, an issue which will be addressed later on.

The clustering properties of dark energy are the subject of recent debate. Even though Caldwell *et al* [13] have shown that Quintessence cannot be perfectly smooth, it is assumed that the clustering is negligible on scales less than 100 Mpc. It is, therefore, a common practice to keep the Quintessence homogeneous during the evolution of the system. On the other hand, one should bear in mind that every positive energy component other than the cosmological constant is capable of clustering, the question is at what rate. Additionally, we are looking at the evolution of the perturbation well beyond the linear regime, where the reaction of the dark energy to the local metric is unclear. Clustering of dark energy is particularly well motivated for models in which dark energy is coupled, in some form, to the matter [10, 14]. The resolution of this debate cannot be found in the prescription of the spherical collapse, Moreover, a top hat profile for the dark energy is not a stable configuration. This work does not attempt to provide an answer to whether and how the dark energy clusters, but rather explores the consequences of such a scenario. A more fundamental treatment of the clustering properties of dark energy is needed, and is a subject of an ongoing investigation [15]. "

The above equations, supplied with the appropriate initial conditions, can now be solved. Following the mathematical solution of the perturbations leads to a decoupling of the perturbation from the background. The local scale factor R evolves in a slower fashion than the global scale factor a, reaches its maximal size R_{ta} at turnaround, and then the system begins to collapse. Following the mathematical solution all the way through leads to a singularity.

III. VIRIALIZATION

Even though the mathematical solution of the spherical collapse equations gives a point singularity as the final state of the system, we know that, physically, objects go through a virialization process, and stabilize towards a finite size. Virialization is not 'built in' into the spherical collapse model (see though [16]), and the common practice is to *define* the virialization radius as the radius at which the virial theorem holds, and the kinetic energy T is related to the potential energy U by

$$T_{vir} = \left(\frac{R}{2}\frac{\partial U}{\partial R}\right)_{vir} \quad . \tag{10}$$

Using energy conservation between virialization and turnaround (where $T_{ta}=0$) gives

$$\left(U+\frac{R}{2}\frac{\partial U}{\partial R}\right)_{vir}=U_{ta}\ . \tag{11}$$

Equation (11) defines R_{vir}.

The fact that the details of the virialization process are bypassed by the above procedure is useful because these details are complex and not fully understood. On the other hand, this means that this procedure cannot provide us with information about how the dark energy behaves during the virialization process. The above energy budget should be applied to *those components that virialize*. If only the matter virializes, equation (11) becomes

$$\left(U(\rho_{mc})+\frac{R}{2}\frac{\partial U(\rho_{mc})}{\partial R}\right)_{vir}=U(\rho_{mc})_{ta}\ , \tag{12}$$

and if the whole system virializes as a whole - both the matter and the Q field, equation (11) should then read

$$\left(U(\rho_{tot})+\frac{R}{2}\frac{\partial U(\rho_{tot})}{\partial R}\right)_{vir}=U(\rho_{tot})_{ta}\ , \tag{13}$$

with $\rho_{tot}=\rho_{mc}+\rho_{Qc}$. As will be shown in the next section, the choice which of the components actually virializes make a significant difference.

Whether the dark energy participates in the virialization is an open question. In principle, any energy component with non vanishing kinetic energy is capable of virializing, given enough time. Once again, our aim is not to settle the issue of the virialization properties of the dark energy, but rather consider and compare the consequences of both options. While it seems reasonable to connect between the clustering and the virializing properties of dark energy, this may lead to some problems, because clustering is a continuous property, while the virialization is not. We will, therefore, consider various degrees of clustering, regardless of whether the dark energy virializes or not.

IV. ENERGY NON CONSERVATION

Equation (13) assumes energy conservation for the virializing component(s) between turnaround and the time of virialization. As was mentioned earlier, the Q field loses energy to the background if $\gamma\Gamma\neq 0$, that is if the dark energy is *not* a cosmological constant, but is

nonetheless not allowed to follow the local metric and cluster. If such a Q field participates in the virialization, then equation (13) needs to be corrected, in order to account for the energy lost.

The continuity equation for the energy losing field is

$$\dot{\rho}_{Qc} + 3(1+w)\left(\frac{\dot{R}}{R}\right)\rho_{Qc} = \gamma\Gamma \ . \tag{14}$$

We can imagine a field with the same equation of state that *does* conserve energy, $\tilde{\rho}_{Qc}$. The continuity equation for this energy-conserving field is then

$$\dot{\tilde{\rho}}_{Qc} + 3(1+w)\left(\frac{\dot{R}}{R}\right)\tilde{\rho}_{Qc} = 0 \ . \tag{15}$$

If we chose the same initial conditions for the real field and for the energy-conserving field at turnaround, $\tilde{\rho}_{Qc}(t_{ta}) = \rho_{Qc}(t_{ta})$, then by construction, the amount of the energy which was lost at a later time is

$$\Delta U \equiv \tilde{U} - U \ , \tag{16}$$

where $\tilde{U}(\rho_{Qc}) \equiv U(\tilde{\rho}_{Qc})$ is the potential energy function for the energy-conserving field. The correction to equation (13) is then

$$\left(U + \Delta U + \frac{R}{2}\frac{\partial U}{\partial R}\right)_{vir} = \left(\tilde{U} + \frac{R}{2}\frac{\partial U}{\partial R}\right)_{vir} = U_{ta} \ . \tag{17}$$

As will be shown in the next section, this introduces a small quantitative correction to the final state of the system.

A different source of energy non conservation (ENC) is when the Q field is kept homogeneous (or does not fully cluster, $\gamma \neq 0$), and does not participate in the virialization. The virializing component then feels a time dependent, non conservative force. This energy non conservation was addressed in [12].

V. RESULTS

Figure 1 gives a summary of the different choices one can make when applying the spherical collapse formalism to a cosmology with dark energy. It shows the dependence

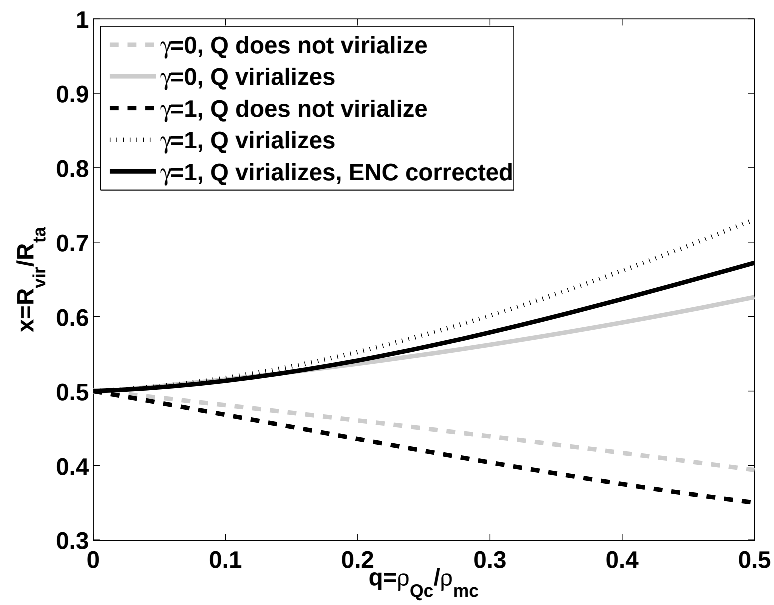

FIG. 1: The ratio of final to turnaround radii, $x = R_{vir}/R_{ta}$, as a function of $q = \rho_{Qc}/\rho_{mc}$ at turnaround, for Quintessence with a constant equation of state $w = -0.8$.

of the final size of an overdense system relative to its maximal size, as a function of the strength of the dark energy at the time of turnaround, $q = (\rho_{Qc}/\rho_{mc})_{ta}$. For all curves the equation of state of the dark energy is $w = -0.8$. The grey lines are for fully clustering dark energy, while the black lines are for the homogeneous case. Dashed lines follow the behavior of the system when only the matter component virializes, and the solid lines are for the case when the complete system virializes as a whole, including the dark energy component. Comparison of the dotted line with the solid black line shows the effect of the ENC correction. As can be seen, all cases converge to $x = 1/2$ as $q \to 0$, which is the analytical result for a universe made strictly of matter. In general, one can say that if only the matter virializes, the presence of dark energy creates bound objects which are smaller and denser. Allowing the dark energy to participate in the virialization results in larger, less dense objects. This effect is enhanced if the dark energy is kept homogeneous. Correcting for the energy loss of a homogeneous dark energy that virializes slightly weakens the effect, but does not introduce any qualitatively new behavior.

The figure shows how the various cases behave as a function of q, which is, in turn, dependent on the cosmological model, as well as on the time in which the turnaround

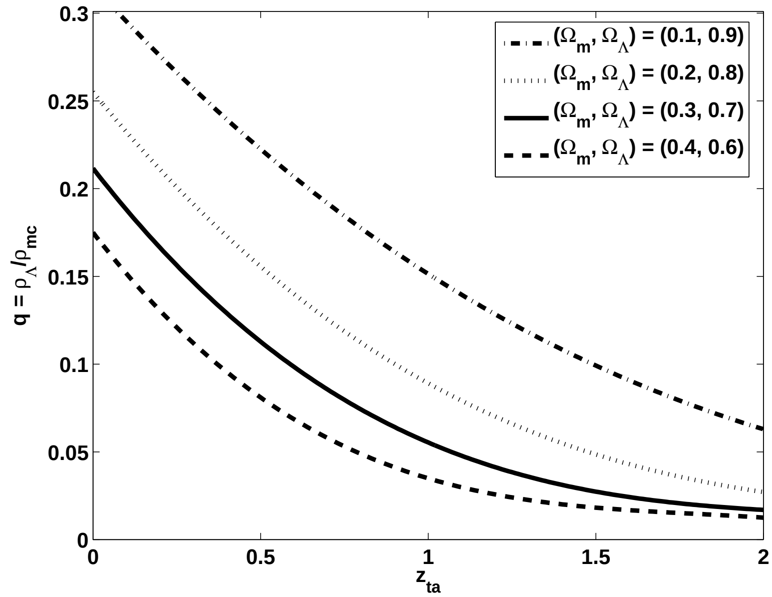

FIG. 2: $q = \rho_\Lambda/\rho_{mc}$ as a function of the turnaround redshift z_{ta}, for various values of Ω_m and Ω_Λ.

happens. Figure 2 shows the dependence of q as a function of the turnaround redshift, for various ΛCDM cosmologies. As the figure shows, typical q values of cosmological interest are less than 0.3.

A better understanding of what the virialization process consists of is needed in order to decide whether dark energy participates in it, and here one can only speculate. The most motivated case for allowing the dark energy to virialize is when it clusters, for the clustering can be interpreted as a sign that the dark energy feels and responds to the local interactions, and possibly also to those that lead to virialization. A natural approach, therefore, would be to associate between the clustering and the virialization processes. This approach poses a problem though, illustrated in figure 3. The figure shows the solutions of x as a function of γ, with fixed $w = -0.8$ and $q = 0.2$. The circle on the right is the Wang and Steinhardt's result when the quintessence is kept completely homogeneous, and only the matter component virializes. The square on the left is the result when the dark energy fully clusters, and both the matter and the dark energy virialize. One would expect the transition in the behavior of the system along γ to be smooth. Allowing the dark energy to virialize for the clustering case, $\gamma = 0$, and keeping it out of the virialization process when $\gamma = 1$, raises the question of how one should extrapolate smoothly between the two cases. As figure 3 suggests, there will be a discontinuity.

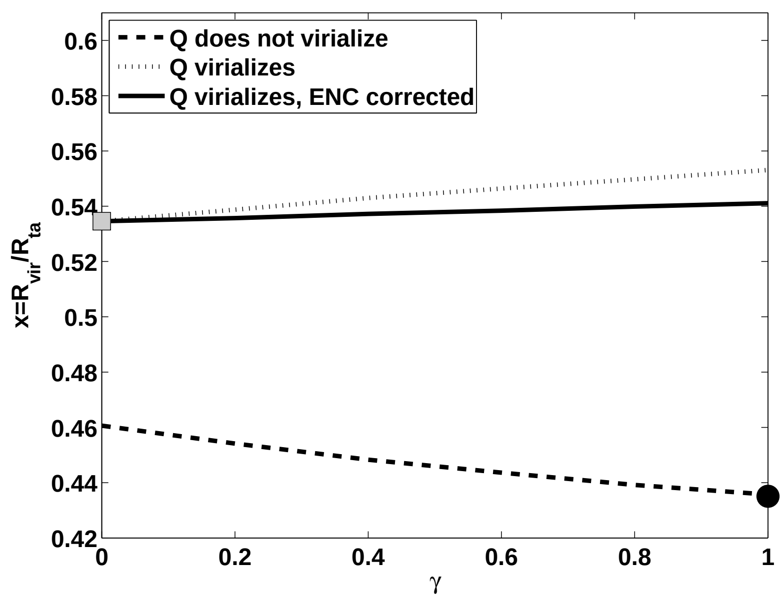

FIG. 3: $x = R_{vir}/R_{ta}$ as a function of the clustering parameter γ, for $w = -0.8$ and $q = 0.2$. $\gamma = 0$ describes the case of a fully clustering Q field, and $\gamma = 1$ is the case of a homogeneous Q, allowing only the matter component to cluster. For $\gamma = 0$, taking the dark energy into the virialization is highly plausible, (see square on left). If one assumes that only the matter component virializes for $\gamma = 1$ (see circle on right), it is unclear how to extrapolate in a smooth way between the two cases. This will produce a discontinuity in the transition from the 'clustering' to the 'non-clustering' behavior.

Another issue which deserves special attention is the limit of the cosmological constant, $w \to -1$. For $w = -1$ the clustering parameter γ plays no role, because the question whether such a fluid is allowed to cluster ($\gamma = 0$) or not ($\gamma = 1$) is rather abstract. It stays homogeneous in any case, because of its equation of state, $w_\Lambda = -1$ (which leads to $\Gamma = 0$). Accordingly, energy is automatically conserved. An equivalent of figure 1 drawn for $w = -1$ will show that the grey and the black dashed lines coincide, as well as the grey and the black solid lines. The dotted line will coincide with the solid ones as well, because $\Delta U = 0$. The difference between dark energy which does or doesn't virialize is still evident though. Again, one should consider the plausibility of the two solutions. If one considers the cosmological constant as a true constant of Nature, $\rho_\Lambda = \Lambda/(8\pi G)$, it is hard to imagine it participating

in the dynamics that lead to virialization, as it is a true constant. In this case, one could categorically say that the right procedure is to look at the virialization of the matter fluid only, following the work of Lahav *et al* [4] (see circle on left in figure 4). The sole effect of the cosmological constant, then, is to modify the potential that the matter feels, through the background expansion.

If, on the other hand, one considers the origin of a perfect fluid with $w \approx -1$ as a special case of quintessence, which is indistinguishable from a cosmological constant, it is reasonable to expect continuity in the behavior of the system as one slowly changes the value of w toward -1. In other words, if the physical interpretation of the fluid with $w \approx -1$ is of a dynamical field that *mimics* a constant, the idea of including it in the dynamics of the system has a physical meaning.

The result, then, is that we possibly have a signature differentiating between a cosmological constant which is a true constant, and a different entity which *mimics* a constant. This point is shown in figure 4. The figure shows x as a function of w, with fixed $q = 0.2$ and $\gamma = 0$. The dashed line shows how x depends on w when only the matter virializes. The circle on the left is Lahav *et al*'s solution for the cosmological constant. The solid line shows how the system behaves when both the matter and the dark energy virialize. The square on the right is an example of a clustered dark energy, where we expect to take into account the whole system in the virialization. As with figure 3, there is a suggested discontinuity, but here one can associate the discontinuity with a clear physical meaning: a true cosmological constant is not on the continuum of perfect fluids with general w, as its physical behavior is different. An observational detection of virialized objects with $R_{vir} > R_{ta}/2$ would be a strong evidence against a cosmological constant which is a true constant, regardless of the measured value of the equation of state.

VI. CONCLUSIONS

The inclusion of dark energy into the spherical collapse formalism was reviewed. As the spherical collapse formalism is not a calculation from first principles, it does not provide information about the behavior of the dark energy during the evolution of an overdensity. Specifically, one needs to decide whether to allow the dark energy to cluster, and whether one should include it in the virialization process. The consequences of these choices were

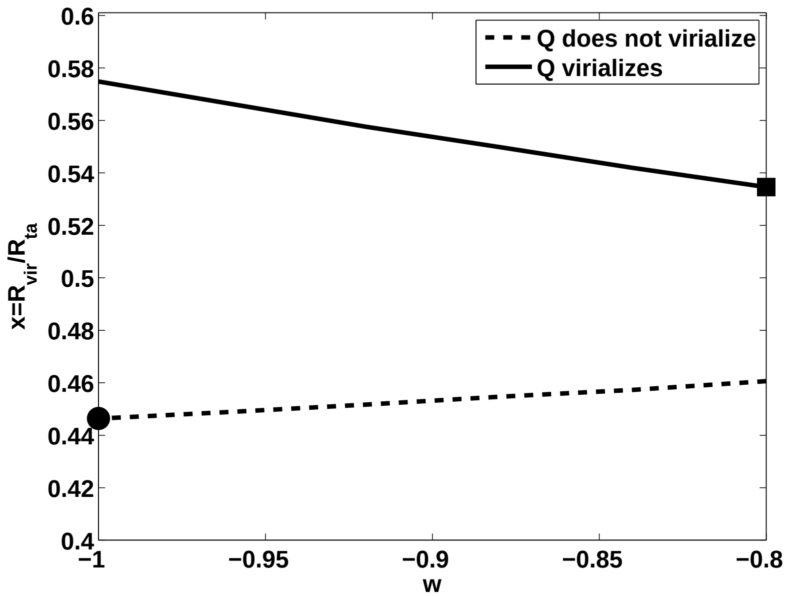

FIG. 4: $x = R_{vir}/R_{ta}$ as a function of the equation of state w, for $q = 0.2$ and $\gamma = 0$. The dashed line is the ratio when the matter alone virializes, and the solid is for the case where the whole system virializes. The circle on the left is Lahav *et al*'s solution for the cosmological constant [4]. The square on the right is an example of a clustered quintessence, where we expect to take into account the whole system in the virialization. The figure suggests we should expect a discontinuity in the behavior of quintessence fields and a true cosmological constant.

examined and are summarized in figure 1. An additional issue which was addressed here is the energy non-conservation, which arises when the dark energy is kept homogeneous and is part of the virialization.

The primary result suggests the possibility of an observational signature differentiating between a true cosmological constant, and a dynamical form of dark energy that mimics a constant. An observational detection of virialized objects with $R_{vir} > R_{ta}/2$ would be a strong evidence against a cosmological constant which is a true constant. This signature is unique in the sense that it does not depend on measuring the value of the equation of state of dark energy, contrary to most existing probes of dark energy. In order to discuss this signature in more definite and quantitative terms, a better understanding of the dark energy behavior during the evolution of overdensities is required. The clustering properties of dark energy [15], as well as a more detailed understanding of how systems virialize,

need to be further explored. This information cannot be found in numerical simulations. The incorporation of dark energy into simulations is done by modifying the background expansion. It means that by construction we will not be able to see clustering or virialization of dark energy. Thus another direction to pursue is how to improve the incorporation of dark energy into simulations.

These directions of future research are particularly important, because observational evidence seems to point that even if the dark energy is in essence dynamical, it is doing a very good imitation of a cosmological constant. As the implications of the two options are so dramatically different, it is worth exhausting all possibilities we can think of to distinguish between a cosmological constant and dynamic dark energy. This is an exiting challenge.

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